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ON DISCRETE DERIVATIVE AND INTEGRALS

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İt is known that many papers have been devoted to the discrete equation and to the problems of such kind equations. A majority of them refer to discrete additive derivative equations. There are few papers devoted to discrete multiplicative derivative equations and the problems for such equations. Finally, its hould be noted that there are few references devoted to the equations containing both discrete additive and discrete multiplicative derivatives and the problems stated for them.

Here, we consider general solution of multiplicative and addive multiplicative derivative equations.

Key words: Discrete additive derivative, discrete multiplicative derivative, discrete additive integral, discrete multiplicative integral.

Introduction

Discrete analysis mainly discredits continuous problems and finds approximate solution for these continuous problems [1], [3], [4]. Notice that, there are some cases when discrete analysis constructs mathematical model of discrete events and solves the obtained mathematical problem.

In this paper, we'll be engaged in discrete additive and discrete multiplicative derivative equations and the problems stated for them.

Our main goal is to construct general theory for the examples considered in [7], [8].

Definition 1. *Discrete additive derivative of the function* $f(x)$ *determined in the given set N or Z or in their subset is determined as follows:*

$$f^{(1)}(x) = f(x+1) - f(x). \tag{1}$$

Definition 2. *Discrete multiplicative derivative of the function* $f(x)$ *given in Definition 1 is determined in the following way:*

$$f^{[1]}(x) = \frac{f(x+1)}{f(x)},\ f(x) \neq 0 \cdot \tag{2}$$

Definition 3. *When we say discrete additive integral of the function*

$f(x)$ *given in definition 1 we understand the expression*

$$\int_n^m f(x) = \sum_{k=n}^{m-1} f(k). \qquad (3)$$

Definition 4. *When we say discrete multiplicative integral of the function* $f(x)$ *given in Definition 1 we mean the expression* [2], [7].

$$\int_n^m f(x) = \prod_{k=n}^{m-1} f(k). \qquad (4)$$

General solution of variable coefficient, linear, ordinary, discrete additive derivative, homogeneous differential equation of higher order

Let's consider the following equation

$$y^{(n)}(x) + a_1(x) y^{(n-1)}(x) + ... + a_n(x) y(x) = 0, \quad x \in J \subset \mathrm{Z}, \qquad (5)$$

here $n \in N$ is the noted number, $a_k(x)$, $k = \overline{1,n}$, $x \in J$ are the given coefficients, $y(x)$ are the unknowns.

We accept the following denotation

$$y(x) = y_1(x), y^{(1)}(x) = y_1^{(1)}(x) = y_2(x),$$
$$y^{(11)}(x) = y_1^{(11)}(x) = y_2^{(1)}(x) = y_3(x)\ldots..,$$
$$y^{(n)}(x) = y_1^{(n)}(x) = y_2^{(n-1)}(x) = ... = y_{n-1}^{(11)}(x) = y_n^{(1)}(x).$$

Then we can write equation (5) in the form of the following homogeneous system of first order

$$\begin{cases} y_1^{(1)}(x) = y_2(x), \\ y_2^{(1)}(x) = y_3(x), \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ y_{n-1}^{(1)}(x) = y_n(x), \\ y_n^{(1)}(x) = -a_n(x) y_1(x) - a_{n-1}(x) y_2(x) - ... - a_1(x) y_n(x) \end{cases} \qquad (6)$$

We can easily see that the matrix notation of system (6) is as follows:

$$Y^{(1)}(x) = A(x) Y(x), x \in J. \qquad (7)$$

Moreover

$$A(x) = \begin{pmatrix} 0 & 1 & 0 & 0 & ... & 0 & 0 \\ 0 & 0 & 1 & 0 & ... & 0 & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & 0 & 0 & ... & 0 & 1 \\ -a_n(x) & -a_{n-1}(x) & -a_{n-2}(x) & -a_{n-3}(x) & ... & -a_2(x) & -a_1(x) \end{pmatrix} \qquad (8)$$

$$Y(x)=(y_1(x)\ y_2(x)....y_n(x))^T. \qquad (9)$$

Now, let's write (7) in the following way:

$$Y(x+1)=[I+A(0)]Y(0). \qquad (10)$$

Hence we easily get:

$$Y(x)=[I+A(x-1)][I+A(x-2)]...[I-A(x)]Y(x),$$

or $$Y(x)=\left(\int_0^x [I+A(\xi)]\right)Y(0), \qquad (11)$$

here in the $\oint$ discrete multiplicative integral the growth index is from the right hand side to the left hand side, $Y(0)$ is any column.

Theorem 1. *If* $a_j(x)$, $j=\overline{1,n}$, $x\in J\subset Z$ *are the given functions, then the solution of (10) is given in the form (11),* $Y(0)$ *is arbitrary.*

Solution of discrete multiplicative derivative ordinary differential equation

Let's consider the following equation [5], [6].

$$\prod_{k=0}^{n}\left(y^{[k]}(x)\right)^{a_k(x)} = f(x), x\in J, \qquad (12)$$

here $a_k(x)$, $k=\overline{0,n}$ and $f(x)$are the known functions, $y(x)$ is an unknown function. If we take the logarithm of this equation we get:

$$\sum_{k=0}^{n} a_k(x)\ln y^{[k]}(x)=\ln f(x), x\in J, \qquad (13)$$

here

$$y^{[k]}(x)=\frac{y(x+k)y^{C_k^2}(x+k-2)....}{y^{C_k^1}(x+k-1)y^{C_k^3}(x+k-3)....}, \qquad (14)$$

$C_k^m=\dfrac{k!}{m!(k-m)!}$ - are binomial coefficients.

Therefore if

$$\begin{aligned}&\ln y^{[k]}(x)=\ln y(x+k)-C_k^1\ln y(x+k-1)+\\&+C_k^2\ln y(x+k-2)...(-1)^k C_k^k\ln y(x)=\\&=\sum_{s=0}^{k}(-1)^s C_k^s \ln y(x+k-s)\end{aligned} \qquad (15)$$

the equation (13) will take the form:

$$\sum_{k=0}^{n} a_k(x)\sum_{s=0}^{k}(-1)^s C_k^s \ln y(x+k-s)=\ln f(x), x\in J,$$

or

$$\sum_{s=0}^{n}\left(\sum_{k=s}^{n} a_k(x)(-1)^s C_k^s\right) z(x+k-s) = \ln f(x), \tag{16}$$

here $z(x+k-s) = \ln y(x+k-s)$.

So we reduced the discrete multiplicative derivative differential equation (12) to the differential equation (16) and as we can reduce it to the equation (5), the solution of this equation is obtained from theorem 1.

Solution of equations containing discrete additive and multiplicative derivative

Let's consider the following equation:

$$\left(y^{[n]}(x)\right)^{(m)} + a(x) y^{[n]}(x) = f(x), x \in J, \tag{17}$$

here $a(x)$ and $f(x)$ are the given functions, $y(x)$ is an unknown.

If we substitute

$$y^{[n]}(x) = Z(x) \tag{18}$$

we get the following discrete additive derivative, variable coefficient, linear equation

$$Z^{(m)}(x) + a(x) Z(x) = f(x). \tag{19}$$

This is a special case of the equation (5). If we write the found $Z(x)$ in (18), we get a special case of the equation (12) and this case has already been solved.

Remark. In the considered equation (17) we accept $n = 3$, $m = 2$, $a(x) = f(x) = 0$ and write it in the open form and get:

$$y(x)y(x+1)y^2(x+2)y^6(x+3)y(x+5) - 2y(x)y^6(x+2)y^4(x+4) +$$
$$+ y^4(x+1)y^4(x+3)y^3(x+4) = 0.$$

Such type equations were considered in [7].

Unsolved problems

- Solve the following problem

$$\left(y^{[11]}(x)\right)^{(1)} + 2\left(y^{(11)}(x)\right)^{[1]} + y(x) = 0, x \ge 0, y(k) = \alpha_k, k = 0,1,2;$$

- Solve the following boundary value problem:

$$\left(y^{[1]}(x)\right)^{(1)} + a\left(y^{[11]}(x)\right)^{(11)} + by(x) = 0, x \in [0, n-2], y(0) = \alpha, y_n = \beta;$$

- Consider the problems for the above mentioned equations under non-local boundary conditions;
- Reduce the above mentioned equations to the problems for higher order equations.

REFERENCES

1. Atkinson F.V. Discrete and Continuous Boundary Problems. Academic Press, New York, London, 1964, 672 p.

2. Riordan J.Combinatorial Identities. New York-London-Sydney, 1968, 245 p.
3.Aliev N., Bagirov G., Izadi F.A. Discrete Additive Analysis. Univ. Tarb. Mual. Tabriz, Iran, 1993, 144 p.
4. Gelfond A.O. Calculus of Finite Differences. M.: Nauka, 1967, 375 p.
5. Aliev N., Azizi N., Jahanshahi M. Invariant Functions for Discrete Derivatives and their Applications to Solve non-Homogeneous Linear and non-Linear Difference Equations // International Mathematical Forum, 2, 2007, No.11, p.533-542.
6. Aliev N., Jahanshahi M. Khatami H.R., Innovation of Discrete Additive and Multiplicative Analysis. Research Project, Azad Islamic University of Karaj, Iran, 2003, 60 p.
7. Hassani Oskauei L. and Aliev N. Analytic Approach to Solve Specific Linear and Nonlinear Difference Equations // International Mathematical Forum, 3, 2008, No. 33, p.1623-1631.
8. Mehdiyev M., Ahmedov R., Eyvazov E., Sharifov Y. Numerical Methods. Baku: Teknur, 2008, 376 p.

DİSKRET TÖRƏMƏ VƏ İNTEQRALLAR HAQQINDA

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XÜLASƏ

Diskret tənliklərə və onlar üçün qoyulmuş məsələlərə kifayət qədər işlər həsr olunmuşdur. Bunların əksəriyyəti diskret additiv törəməli tənliklərə aiddir. Diskret multiplikativ törəməli tənliklərə və onlar üçün qoyulmuş məsələlərə kafi qədər az işlər həsr olunmuşdur. Nəhayət qeyd edək ki, həm diskret additiv, həm də diskret multiplikativ törəmələr tutan tənliklər və onlar üçün qoyulmuş məsələlərə ədəbiyyatda çox az təsadüf olunur. Biz burada multiplikativ və multipliko-additiv törəməli tənliklərin ümumi həllinin tapılması ilə məşğul olmuşuq.

Açar sözlər: diskret additiv törəmə, diskret multiplikativ törəmə, diskret additiv inteqral, diskret multiplikativ inteqral.

О ДИСКРЕТНОЙ ПРОИЗВОДНОЙ И ИНТЕГРАЛА

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РЕЗЮМЕ

Известно, что как дискретному уравнению, так и задаче такого рода уравнений посвящена многочисленная работа. Многие из них относятся к дискретному аддитивному произвольному. Дискретное мультипликативное производное и задачи для таких уравнений рассмотрены очень мало. Наконец, заметим, что дискретное аддитивное и дискретно мультипликативное производные, а также задачи для таких уравнений рассмотрены незначительно. Мы рассматриваем обучение решение дискретную и мультипликативную производную и уравнений дискретно мультиплико - оддитивной производной.

Ключевые слова: дискретное аддитивное производное, дискретное мультипликативное производное, дискретный аддитивный интеграл, дискретный мультипликативный интеграл.

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